

of these assumptions seems too likely, and indeed the true thermal condition probably lies somewhere between these two limiting states.

In this paper we present equations to represent the composite as a classical mixture. We shall show that the theory of Tsou and Chou can be obtained as a special case of the more general mixture theory, and that the mixture approach will yield the necessary tools to consider phase transformations, chemical reactions, and partial heat transfer among the constituents within the shock surface. The mixture theory approach has previously been employed by Riney, *et. al.* [7] and Davis [8] to represent geologic materials subject to shock loading. A large volume of work concerning applications of mixture theory to other types of loading for various materials has also appeared (e.g., Knops and Steel [9]).

### BASIC EQUATIONS

Mixture theory is founded on a set of concepts elucidated by Truesdell and Toupin [10]. Each constituent of the mixture is considered a separate continuum which may be acted upon by exterior actions and by other constituents. It is postulated that the mean motion of the whole mixture is a mathematical consequence of the motions of the individual constituents and, moreover, that this mean motion must obey the laws which govern the motion of a single material. Thus, the mixture as a whole does not know whether it is composed of only one or of several constituents. Since each constituent is considered a continuum, every place in the mixture may be simultaneously occupied by particles of any or all of the various constituents. Whereas mass, momentum, and energy of each constituent are not necessarily conserved, the mixture mass, momentum, and energy are.

We shall designate the mixture constituents by  $S_\alpha$ ,  $\alpha = 1, 2, \dots, k$ . The total number of constituents,  $k$ , is left arbitrary. By  $\rho_\alpha$  we denote the mass density of  $S_\alpha$ . Since  $S_\alpha$  is a continuum,  $\rho_\alpha$  is a bulk density; that is, the mass of  $S_\alpha$  per unit volume of the mixture. The mixture density, designated  $\rho$ , is given by

$$\rho = \sum_{\alpha} \rho_{\alpha} \quad (1)$$

Henceforth, quantities lacking Greek subscripts will refer to the mean properties of the mixture, and the symbol  $\sum_{\alpha}$  will always indicate the summation over  $\alpha = 1, 2, \dots, k$ .

The particle velocity of the constituent  $S_\alpha$  is given by

$$\mathbf{v}_{\alpha} = \left. \frac{\partial \mathbf{x}_{\alpha}}{\partial t} \right|_{x_{\alpha} = \text{constant}} \quad (2)$$

Here  $\mathbf{x}_\alpha$  is the place occupied by the particle  $X_\alpha$  at time  $t$ . We also introduce the concentration or mass fraction,  $c_\alpha$ , for the constituent  $S_\alpha$ .

$$c_\alpha = \rho_\alpha / \rho \quad (3)$$

Then the mean velocity of the mixture,  $\mathbf{v}$ , is given by the mass-weighted average of the individual velocities.

$$\mathbf{v} = \sum_{\alpha} c_{\alpha} \mathbf{v}_{\alpha} \quad (4)$$

We also define the diffusion velocity,  $\mathbf{u}_\alpha$ , of the constituents  $S_\alpha$  as

$$\mathbf{u}_\alpha = \mathbf{v}_\alpha - \mathbf{v} \quad (5)$$

Multiplying Equation (5) by  $\rho_\alpha$  and summing over  $\alpha$  leads to the identity

$$\sum_{\alpha} \rho_{\alpha} \mathbf{u}_{\alpha} \equiv \mathbf{0} \quad (6)$$

We next define a partial stress tensor,  $\mathbf{T}_\alpha$ , for the constituent  $S_\alpha$ , as well as a specific internal energy,  $\epsilon_\alpha$ , and a heat flux vector,  $\mathbf{h}_\alpha$ . The total stress,  $\mathbf{T}$ , internal energy,  $\epsilon$ , and heat flux,  $\mathbf{h}$ , for the mixture are then given by the following relations.

$$\mathbf{T} = \sum_{\alpha} (\mathbf{T}_\alpha - \rho_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha) \quad (7)$$

$$\epsilon = \sum_{\alpha} c_{\alpha} \left( \epsilon_{\alpha} + \frac{1}{2} u_{\alpha}^2 \right) \quad (8)$$

$$\mathbf{h} = \sum_{\alpha} \left\{ \mathbf{h}_{\alpha} + \mathbf{T}_{\alpha} \mathbf{u}_{\alpha} - \rho_{\alpha} \left( \epsilon_{\alpha} + \frac{1}{2} u_{\alpha}^2 \right) \mathbf{u}_{\alpha} \right\} \quad (9)$$

In the above equations,  $\otimes$  denotes the tensor product of two vectors, while  $u_\alpha^2$  denotes the inner product of  $\mathbf{u}_\alpha$  with itself. Motivation for Equations (7), (8), and (9) may be found in Reference [10].

We shall now set down three equations which govern the balance of mass, momentum, and energy for each of the mixture constituents,  $S_\alpha$ , crossing a shock surface. These relations were first derived by Kelly in Reference [1]. We denote by  $\mathbf{U}$  the spatial velocity of the shock, assumed normal to the shock surface. Letting  $\Psi$  represent any quantity which approaches continuous limits on both sides of the shock surface, we denote the jump in  $\Psi$  by

$$[\Psi] = \Psi^+ - \Psi^-$$

where  $\Psi^+$  represents the limiting value of  $\Psi$  immediately in front of the shock and  $\Psi^-$  the value immediately behind.

The balance of mass jump relation for the constituent  $S_\alpha$  crossing the